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PAR-003-1015044

Seat No. ________

B. Sc. (Sem. V) (CBCS) (W.I.F. 2016) Examination

October / November - 2018

Statistics : S - 503

(Statistical Inference)

Faculty Code : 003

Subject Code : 1015044

Time : **2:30** Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) All questions carry equal marks.

(3) Student can use their own scientific calculator.

1 (a) Give the answer of following question : **4**

(1) The difference between the expected value of an estimator and the value of the corresponding parameters is known as ____________.

(2) If an estimator T_n converges in probability to the parametric function $\tau(\theta)$, T_n is said to be a ____________ estimator.

(3) If the expected value of an estimator T_n is equal to the value of the parameter θ, T_n is said to be ____________ estimator of θ.

(4) An estimator T_n which is most concentrated about parameter θ is the ____________ estimator.

(b) Write any **one** : **2**

(1) Obtain an unbiased estimator of θ for Poisson distribution.

(2) Obtain sufficient estimator of θ for $f(x, \theta) = \theta e^{-\theta x}$ where $0 \le x \le \infty$.

(c) Write any **one** : **3**

(1) Obtain consistent estimator of μ for Normal distribution.

(2) If $t'' = \frac{1}{2}(t + t')$ where t and t' is the most efficient estimator with variance v then prove that $Var(t'') = \frac{1}{2}v\left(1 + \sqrt{e}\right)$.

(d) Write any **one** : **5**

(1) If $x \sim N\left(\mu, \sigma^2\right)$ and μ is known then obtain unbiased estimator of σ.

(2) If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2, σ_2^2 and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such combination ?

2 (a) Give the answer of following question : **4**

(1) If T_1 and T_2 are two MVU estimator for $T(\theta)$, then ____________.

(2) An estimator of $v_\theta\left(T_n\right)$ which attains lower bound for all θ is known as ____________.

(3) An unbiased and complete statistic is a ____________ estimator provided MVUE exists.

(4) For discrete variable Crammer-Rao inequality ____________.

(b) Write any **one** : **2**

(1) Obtain MVUE of parameter p for Binomial distribution.

(2) Define Minimum Variance Bound Estimator.

(c) Write any **one** : **3**

(1) Obtain MVUE and MVBE of θ for Poisson distribution.

(2) Independent observation $x_1, x_2, x_3, \ldots\ldots, x_n$ taken from the following density function

$$f(x, \theta) = \frac{1}{\theta} e^{-\left(\frac{x}{\theta}\right)} \; ; \text{ where } \; 0 \le x \le \infty$$

Find the Cramer Rao Lower Bound for variance of unbiased estimator of θ.

(d) Write any **one** : **5**

(1) State and prove Cramer-Rao Inequality.

(2) Obtain MVUE and MVBE of σ^2 for Normal distribution.

3 (a) Give the answer of following question : **4**

(1) A value of a parameter θ which maximize the likelihood function is known as ____________ estimate θ.

(2) Maximum likelihood estimate of the parameter θ of the distribution $f(x, \theta) = \frac{1}{2} e^{-|x - \theta|}$ is ____________.

(3) For a Gama (x, α, λ) distribution with λ known, the maximum likelihood estimate of α is ____________.

(4) Minimum Chi-square estimators are not necessarily ____________.

(b) Write any **one** : **2**

(1) Obtain likelihood function of Negative Binomial distribution.

(2) Estimate parameter θ by the method of moment for the following distribution $f(x, \theta) = \frac{1}{\theta} x$; where $0 \le x \le 1$.

(c) Write any **one** : **3**

(1) Obtain MLE of parameter p for the distribution $f(x, p) = pq^x$; where $x = 0, 1, 2, \ldots\ldots, \infty$.

(2) Estimate parameters α and β by the method of moment for the following distribution

$$f(x, \alpha, \beta) = \frac{\alpha^{\beta}}{\overline{|\beta}} e^{-\alpha x} x^{\beta-1}; \text{ where } x \geq 0, \alpha > 0.$$

(d) Write any **one** **:5**

(1) Obtain MLE of α and λ for the following distribution

$$f(x, \alpha, \lambda) = \frac{1}{\overline{|\lambda}} \left(\frac{\lambda}{\alpha}\right)^{\lambda} e^{-\left(\frac{\lambda}{\alpha}\right)x} x^{\lambda-1}; \text{ where } 0 \leq x \leq \infty, \lambda > 0$$

where $\varphi(\lambda) = \frac{\partial}{\partial\lambda} \log \overline{|\lambda} = \log\lambda - \frac{1}{2\lambda}$ thus,

$\varphi'(\lambda) = \frac{1}{\lambda} + \frac{1}{2\lambda^2}$.

(2) For the double Poisson distribution

$$P(X = x) = \frac{1}{2} \frac{e^{-m_1} m_1{}^x}{x!} + \frac{1}{2} \frac{e^{-m_2} m_2{}^x}{x!}; 0, 1, 2, \ldots.$$

Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - (\mu_1')^2}$.

4 (a) Give the answer of following question : **4**

(1) Accepting H_0 when H_0 is false is ____________ error.

(2) Probability of ____________ error is called level of significance.

(3) If β is the probability of type II error, the power of the test is ____________.

(4) A null hypothesis is rejected if the value of a test statistics lies in the ____________.

(b) Write any **one** : **2**

(1) Define MP test.

(2) Given a random sample $x_1, x_2, x_3, ..., x_n$ from the distribution with pdf $f(x, \theta) = \frac{1}{\theta}$; where $0 \le x \le \theta$. Obtain power of the test for testing $H_0 : \theta = 1.5$ against $H_1 : \theta = 2.5$ where $c = \{x : x \ge 0.8\}$.

(c) Write any **one** : **3**

(1) Give a random sample $x_1, x_2, x_3, ..., x_n$ from distribution with p.d.f.

$f(x;\theta) = \theta e^{-\theta x}; 0 \le x \le \infty, \theta > 0$

Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(2) Let be the probability that coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type-I error, type-II error and power of test.

(d) Write any **one** : **5**

(1) State and prove the Neyman-Pearson Lemma.

(2) Use Neyman-Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ in the case of Normal distribution $N(\theta, \sigma^2)$ where σ^2 is known.

5 (a) Give the answer of following question : **4**

(1) Likelihood Ratio test for testing a hypothesis, simple or composite, against a __________ or __________ alternative hypothesis.

(2) Likelihood Ratio test is relation the maximum __________ estimates.

(3) To decide about H_0, SPRT involves __________ regions.

(4) The decision criteria in SPRT depends on the function of __________ and __________ errors.

(b) Write any **one** : **2**

(1) Define UMP test.

(2) Define ASN function of SPRT.

(c) Write any **one** : **3**

(1) Construct SPRT of Binomial distribution for testing $H_0 : \theta = p_0$ against $H_1 : \theta = p_1 \left(> p_0\right)$.

(2) Write the properties of Likelihood Ratio test.

(d) Write any **one** : **5**

(1) Let sample distribution $x_1, x_2, x_3, ..., x_n$ taken from

$$f(x, \theta) = \frac{1}{\sqrt{2\pi}} e^{-\left\{\frac{(x-\theta)^2}{2}\right\}} \text{ where } -\infty \leq x \leq \infty, -\infty \leq \theta \leq \infty$$

To Likelihood Ratio $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.

(2) Give the SPRT for test $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 \left(> \lambda_0\right)$, in sampling from a Poisson distribution. Also obtain its OC and ASN function.